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FEASIBILITY STUDY OF AN

ATMOSPHERIC ANALYSIS EXPERIMENT

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1.   INTRODUCTION

In the anticipated analysis of the atmosphere of Mars
with an instrumented entry craft, one of the important tasks
is to develop a method of acquiring a continuous sample of
the atmosphere in a way which assures that all constituents
of the atmosphere reach the analyzer with their original
relative abundance preserved and that the sampling apparatus
makes no alteration of the molecular structure of any of
these constituents or introduces any spurious species.
This report presents the results of an investigation of the
feasibility of an unscattered molecular beam probe sampling
system.

2. DISCUSSION OF METHODS

There are a number of conceptual approaches that may be taken in the basic design of an atmospheric sampler for a high velocity entry craft.

One of these is the conventional ram pressure probe, with or without a limiting orifice. In this approach the high velocity flux incident on the probe entrance is admitted to a cavity where the gas is equilibrated and rethermalized to the temperature of the walls of the cavity. A fraction of this gas is subsequently admitted to the analyzer. It is obvious that in this system, and similarly in any equilibrium system, the high energy molecular flux collides with the surface of the apparatus repeatedly as it cools to the wall temperature. If the accommodation coefficient is high there will be only a few high energy collisions per molecule and if it is low there will be many high energy gas-wall collisions. The actual value of the accommodation coefficient makes little difference in an equilibrium system since all the gas eventually admitted to the analyzer has experienced a sufficient number of high energy collisions with the wall to equilibrate with the wall. At least the first, and perhaps the first few of these collisions are sufficiently energetic to alter the molecular structure since the collision

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energy is above the dissociation energy of some molecular structures and above the excitation energy of many. Also these collisions are sufficiently energetic to desorb all gases physically adsorbed on the surface and many species that are chemically adsorbed. (For a CO_2 atmosphere and an entry craft velocity of 5 Km/sec, the kinetic temperature of the gas relative to the craft is about 45,000°K.) After equilibration with the wall certain gas species may condense or be adsorbed on the wall, either to remain and thus spuriously lower the relative abundance of these species admitted to the analyzer or to be desorbed later by bombardment and an increasing wall temperature and thus spuriously increase the relative abundance of these species admitted to the analyzer.

The exit velocity of the diffuse gas in the cavity is small by comparison with the velocity of the inlet flux, since the mean thermal velocity of the diffuse gas is much smaller than the entry craft velocity. This effect yields a gas composition in the equilibration volume which is a time average over the interval of time corresponding to the exhaust time constant of the cavity (the time required for a given sample of gas in the cavity to diffuse back out the inlet probe). The gas sample admitted to the analyzer therefore is not related to the local atmospheric composition, but rather to the average composition of the atmosphere thru which the entry craft has recently passed.

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Consider a system in which a serious attempt is made
to pump the equilibration cavity. The response time of the
pumped cavity must be sufficiently short that the sample of
gas admitted to the analyzer at any instant of time is, in
fact, a sample of the local atmosphere at the instantaneous
height of the entry craft. The magnitude of this problem
may be appreciated from the following illustration. Suppose
the equilibration cavity has a volume of the order of 1
liter and suppose that the maximum allowed error in height
assigned to an instantaneous gas sample is of the order of
1 kilometer. The entry craft travels a distance of 1 km in
about 0.2 sec., minimum. For the gas composition in the
equilibration volume to accurately represent a composition
average over a height of 1 km, the decade time constant of
the cavity-pump combination must be of the order of 0.1 sec.
This implies that the probe tube and equilibration volume
may not be allowed to load up with gas such that the mean
free path is short compared with the total length of the
probe plus equilibration volume, since under this condition
the gas entering the probe could reach the analyzer by vis-
cous diffusion only, which for a reasonable length probe
would require much more than the 0.1 sec. available. It is
therefore necessary to pump the equilibration volume such

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that the equilibrium gas density is maintained sufficiently low that the mean free path is greater than the probe length. For a probe length of the order of a few inches (say, 10 cm) the pressure must be less than 10^{-3} Torr. Assuming a probe diameter of the order of 0.25 inch and supposing it is required to obtain an atmospheric sample down to 40 ^{K}m above the surface of Mars, it turns out that the required pumping speed for the equilibration volume is of the order of 5000 liters/sec.

There is a rather obvious way to circumvent the pump problem, although it solves none of the other problems mentioned above. Since the entry craft velocity is very much greater than the mean thermal velocity of the undisturbed atmosphere, the installation of an aft facing (very large) exhaust port is equivalent to a nearly perfect pump having the same speed as the port speed (until the entry craft velocity approaches the mean thermal velocity of the undisturbed atmosphere).

In an effort to also solve some of the problems mentioned previously along with the pump problem, the exhaust port concept may be extended until the entire analysis system is exposed. Unfortunately, it appears that there is no practical goemetrical configuration for such an

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instrument that does not also suffer from many of the defects discussed previously. In particular, it is obvious that gas will scatter off the electrodes and support structure of the instrument into (or thru) the ionization region. Also, molecular bombardment and an increasing electrode temperature will liberate gases which tend to move along with the instrument, drifting into the ionization region and introducing spurious species in the composition measurement.

All the above concepts suffer from two common problems: shock waves and aerodynamic heating. Their size is sufficiently large that continuum flow effects, such as shock waves and pressure gradients begin to appear at relatively low densities (high altitudes). For example, a typical size conventional pressure probe would encounter trouble somewhere between 75 and 100 km above the surface of Mars and an exposed analyzer, because of its somewhat larger dimensions would encounter trouble at even higher altitudes. The molecular composition of a sample obtained behind a shock wave is, of course, only remotely related to the molecular composition of the undisturbed atmosphere. The altitude at which shocks begin to appear can only be lowered by reducing the size of the sampling system (or

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of course, the entry craft velocity). Aerodynamic heating
or more precisely, gas enthalpy heating cannot be substan-
tially reduced (except, of course, by reducing the entry
craft velocity). However, in the system described below
the heat input rate is about as low as it is possible to
achieve. Further, the size of the sampler is sufficiently
small that the appearance of shock waves is substantially
delayed, at least at those points where they cannot be tol-
erated (the gas inlet, for example).

3. UNSCATTERED MOLECULAR BEAM SAMPLER

The remainder of this report describes and analyzes
an atmospheric sampler which delivers to the analyzer an
unscattered molecular beam. It is obvious that an unscat-
tered beam (molecular flux which has encountered no collis-
ions) must necessarily be an exact sample of the undisturbed
atmosphere. It is perhaps, more complicated to analyze
than other samplers but it either avoids or minimizes all
the problems discussed previously. The sampler integrates
naturally, into a complete atmospheric analysis experiment
for use on an atmospheric entry craft (see schematic below).

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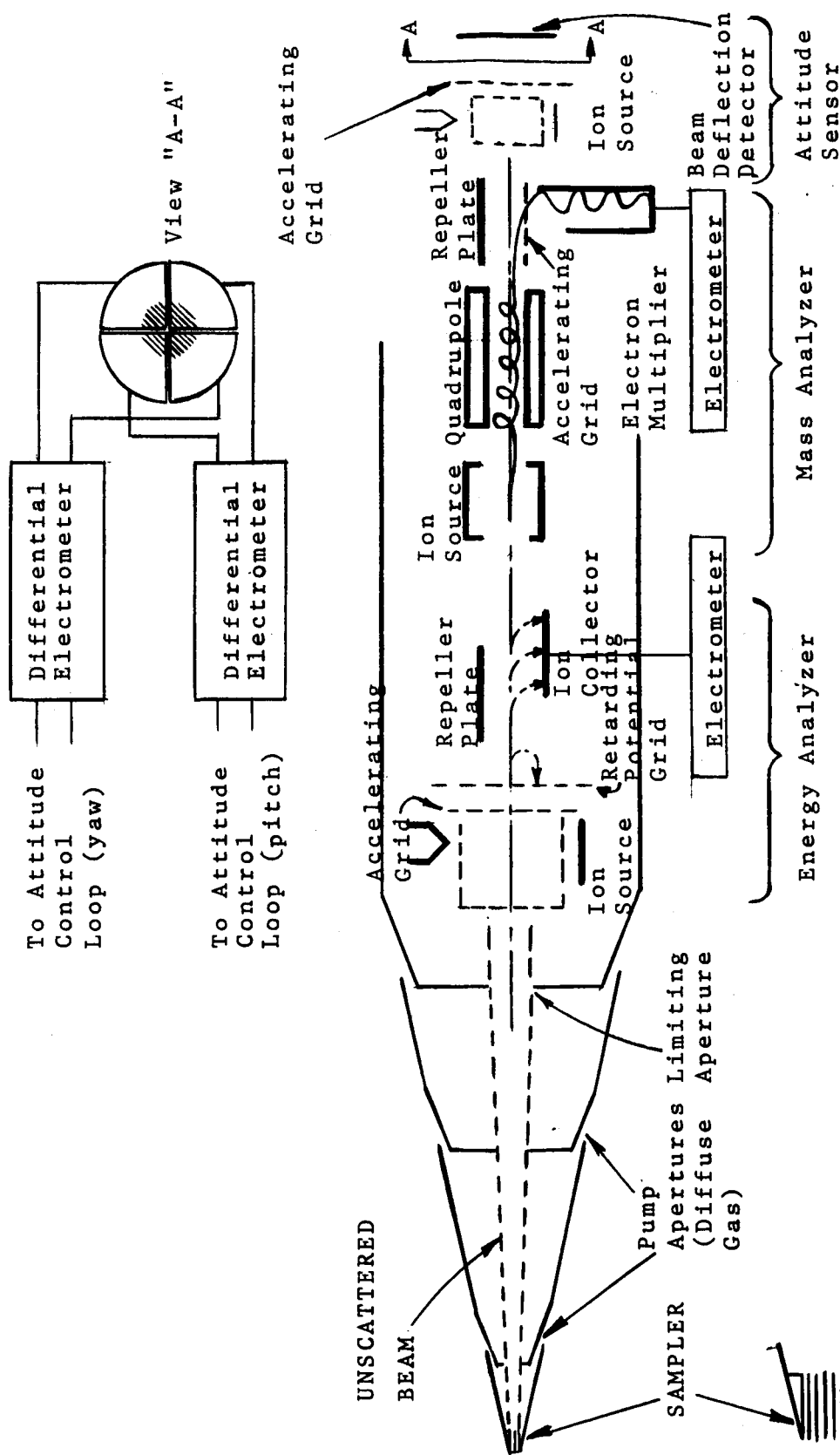
The sampler consists of a parallel bundle of small
diameter tubes having a moderately large length to diameter
ratio, mounted in the end of a support tube. The sampler
tubes have a small diameter and the gas transparency of
the bundle is high to delay (during atmospheric entry) the
appearance of continuum flow effects (such as shocks and
pressure gradients) to the minimum height possible. The
high transparency also minimizes gas enthalpy heating. The
tubes are moderately long to yield a well directed beam and
provide structural stability.

At this point it is considered appropriate to digress
for a moment and give a brief description of the other sub-
systems illustrated in the sketch above.

The sampler support structure is provided with several
aft facing pumping ports to allow the gas which accumulates
in the support tube to diffuse overboard. Several limiting
apertures separate the sampler subsystem from the measure-
ment subsystems. The location and diameter of these aper-
tures are such that most of the unscattered beam from the
sampler passes into the analyzers but only a small fraction
of the diffuse gas reaches the analyzers.

The unscattered beam first passes through the energy
analyzer. A fraction of the beam is ionized. The ions are
extracted and then drift into a retarding potential region.

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Those that have sufficient energy to pass thru the retard-
ing grid are deflected out of the beam and collected. By
varying the effective retarding potential from zero to some
sufficiently large value while measuring the ion current
corresponding to those ions that have sufficient energy to
pass thru the retarding region, the energy distribution of
the molecular flux may be determined. From this data the
thermal velocity distribution or temperature of the undis-
turbed atmosphere may be determined.

The unscattered beam then passes into the ionization
region of the mass analyzer where some fraction of the beam
is ionized. These ions are then extracted from the ion
source and mass analyzed. The output of the mass analyzer
is collected by an off-axis electron multiplier and the
abundance of each species determined.

The beam finally passes into the ion source of the
attitude sensor. The extracted ion current is collected
on a circular disc-target consisting of four separate quad-
rants. The differences in the ion currents collected in
opposite quadrants are measured with differential electro-
meters. If the entry craft velocity vector coincides with
the beam reference axis, the ion currents collected in each
quadrant are the same and the outputs of both differential
electrometers are zero. As the entry craft pitches or yaws

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the unscattered beam no longer coincides with the reference
axis and the ion current collected in one quadrant increases
while in the opposite quadrant decreases. The resulting
differential electrometer output, which is proportional to
the beam deflection angle (pitch or yaw), is applied to
the input of the attitude control loop as an attitude
error signal calling for an attitude control torque in the
direction to realign the attitude of the entry craft.

The unscattered beam may be time-shared or all measure-
ments may be made continuously. Each mode of operation
has particular advantages. The choice of mode of operation
will depend on such parameters as entry craft speed, altitude
resolution required in successive composition measurements
of the atmosphere and the response time of the electronics.

Resuming a discussion of the sampler, it is convenient to
describe the flow phenomena in terms of the collision processes
which an incoming gas molecule encounters within a single tube.
Since the tube bundle diameter is very much smaller than the
sampler-to-analyzer distance, the sampler occupies a very
small solid angle as seen from the analyzer. Further the un-
scattered exit fluxes from the tubes do not interact with
each other since their molecular densities are relatively low,
their "static" temperature is low and the time of flight is
very short. Therefore the total flux reaching the analyzer
is simply the sum of the exit fluxes from all the tubes.

A fraction of the high velocity gas flux incident on the entrance of the tube is transmitted thru the tube without colliding with the tube wall, the remainder collides with the tube wall where it is assumed (in the analytical model used here) to accommodate to the wall temperature and is then reemitted as a diffuse gas. There are therefore two components of the gas flux thru the tube: one consists of that part of the high velocity inlet flux which is transmitted directly thru the tube, and the other consists of that part of the inlet flux which collides with the wall and is converted to a relatively low velocity diffuse gas which then diffuses out of the tube. The fraction of the flux incident upon the tube entrance which is directly transmitted is only a function of the speed ratio (entry craft speed/most probable thermal speed in undisturbed atmosphere) and length to diameter ratio. (Strictly, it is also a function of the angle of attack but for small angles of attack (near zero) the dependance is rather weak. For purposes of clarity, only a zero angle of attack is considered here.) For high speed ratios, the directly transmitted flux is the major fraction of the incident flux. If the flux density incident on the tube entrance is sufficiently low, the fraction of this flux which is scattered off the tube wall and is thus converted to a diffuse gas flux, produces

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a relatively low density diffuse gas within the tube. Under
this condition substantially all the directly transmitted
flux passes thru the tube without colliding with diffuse gas
molecules in the tube. As the inlet flux increases (as the
undisturbed atmospheric density increases) the magnitude of
both the directly transmitted flux and the diffuse flux in
the tube increases. The directly transmitted flux must now
pass thru a tube within which the gas density is higher.
This implies a higher probability that a directly transmit-
ted molecule will collide with a diffuse gas molecule before
it has moved out of the tube. This solution to the sampler
problem is therefore restricted to an atmospheric density
range such that a useful fraction of the directly transmit-
ted flux exits from the tube without having been scattered
by the diffuse gas in the tube.

This exit beam clearly consists of molecules which have
encountered no gas-wall and no gas-gas collision. It is
this unscattered beam which exactly corresponds to the un-
disturbed atmosphere so long as the diffuse gas density in
the tube is sufficiently low that only a small fraction of
the directly transmitted beam is lost by gas-gas scattering.
A mass analysis of this unscattered beam (while the beam
remains in-flight) therefore must yield an exact definition
of the undisturbed atmosphere. As the diffuse gas density

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in the tube increases the unscattered beam flux decreases. Eventually, a sufficiently large fraction of the directly transmitted flux is scattered out of the exit beam and thus the beam flux is no longer related, with precision, to the undisturbed atmospheric density. This defines the minimum height (maximum density) at which the sampler is operable.

Therefore, the fundamental tasks involved in demonstrating that this sampler concept is practicable, are: to derive an exact relation between the unscattered exit flux from the sampler (the beam which enters the analyzer) and is subsequently analyzed and the molecular density in the undisturbed atmosphere ahead of the entry craft; to establish the molecular density of the undisturbed atmosphere, above which gas-gas scattering within the tube is no longer negligable (scattering of the transmitted beam by the diffuse gas within the tube); to show that only a negligable fraction of the diffuse gas effusing thru the tube exit can reach the analyzer; to establish the molecular density of the undisturbed atmosphere above which continuum flow effects are no longer negligable in the operation of the sampler; and from the upper limit of the gas enthalpy flux incident on the sampler, establish the molecular density of the undisturbed atmosphere above which the resulting sampler temperature exceeds its useful limit.

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The results of investigations into these problems are presented in subsequent sections of this report.

3.1 Flow Analysis

Consider a single tube of length L and radius r and suppose that the tube is moving thru the gas with a velocity u, in a direction parallel to the tube axis. The gas molecules incident on the tube entrance may come from anywhere within the half-space in front of the tube. However, for a gas molecule to pass thru the tube, its total velocity vector relative to the tube (in a reference system moving with the tube) must be directed into the conical space shown in the illustration below. In this reference system, the total molecular velocity is the vector sum of the tube velocity and the molecular thermal velocity. Thus, only those molecules which are incident on the tube entrance have any possibility of passing thru the tube. But only a fraction of these molecules can actually pass completely thru the tube without colliding with the tube wall. This fraction consists of molecules which have total velocity components such that $\phi < \tan^{-1}\left(\frac{2r}{x}\right)$. Some typical sets of molecular velocity

components are shown in the illustration below.
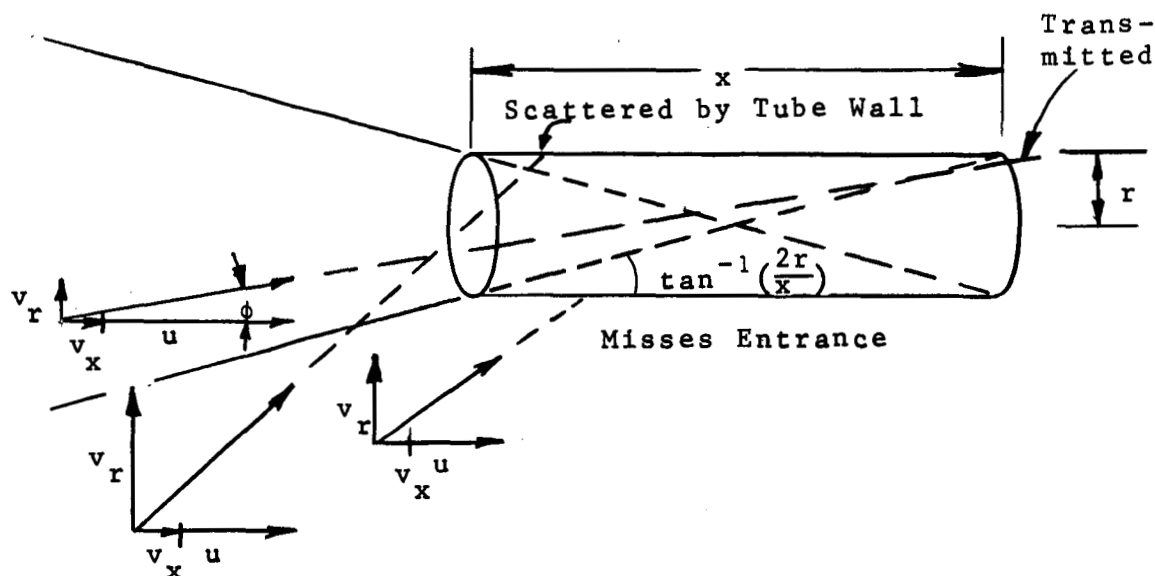
Obviously, the molecules which are incident on the tube
entrance which are not transmitted thru the tube, must
collide with the inside wall of the tube.

The fraction of the flux incident on the tube entrance
which is transmitted thru the tube depends on $\frac{x}{2r}$, the
length to diameter ratio, and on the speed ratio $\frac{u}{v_m}$, where
u is the probe speed and v_m is the most probable molecular
speed (thermal) in the undisturbed atmosphere.

As the speed ratio increases the fraction of the flux

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incident on the tube entrance which is transmitted thru the
tube increases, since at the higher speed ratio the molecules
pass thru the tube in a smaller time interval and thus have
less time to travel across the tube and strike the wall.
This is illustrated in the sketch below.
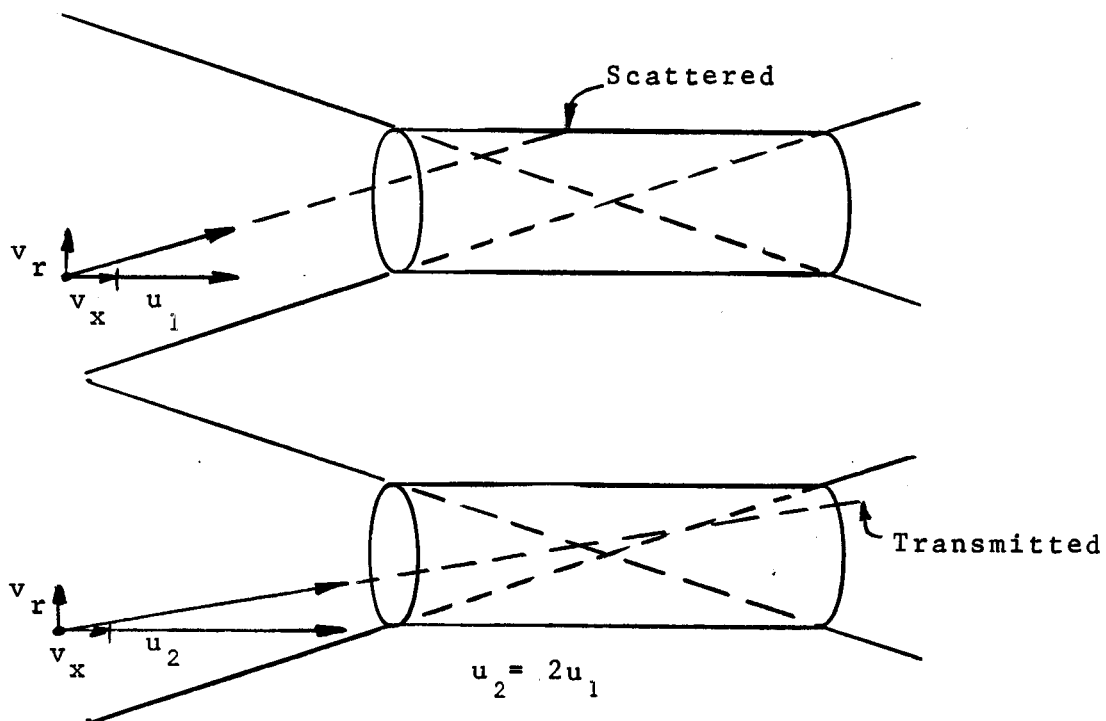
For a fixed speed ratio (of any value) the fraction of the
flux incident on the tube entrance which is transmitted, in-
creases as the length to diameter ratio decreases and as the
tube approaches the geometrical configuration of an aperture,

all the flux incident on the entrance is transmitted. For
relatively high speed ratios, the molecular flux incident
on the inside wall of the tube, collides with the surface
at a relatively high velocity (on the average). The mole-
cules lose some energy in the collisions with the wall.
However a theoretical description has not yet been devel-
oped for the molecule-wall collision process which correctly
accounts for the energy loss in the collision (according
to existing experimental data). In order to proceed beyond
this point it is necessary to make some assumption about
the energy distribution of the molecules reflected from
the wall. The most conservative assumption that can be
made is, that the molecules are diffusely reflected from
the wall with a mean thermal velocity corresponding to the
wall temperature. This assumption has two important advan-
tages: 1) Considering the possible assumptions which are
somewhat realistic, this one is the most conservative in
the sense that it underestimates the maximum density at
which the probe will function since it leads to the maxi-
mum possible value for the density of the diffuse gas within
the tube; 2) This assumption substantially reduces the com-
plexity of the necessary computations encountered in proceed-
ing beyond this point. In fact, it is about the only assump-
tion (with one possible exception) which permits any substantial
analytical progress beyond this point.

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Proceeding under the diffuse reflection assumption, the gas flux incident on the tube wall (inner surface) is completely accommodated at the wall and is reemitted with a Maxwellian velocity distribution corresponding to the wall temperature, and the reemitted flux angular distribution is symmetric about the local surface normal and the flux density (flux/unit solid angle) is proportional to $\cos \theta$, where θ is the angle of emmission measured from the normal. Thus, the tube wall acts as a sink for the incident flux of high velocity molecules, and as a source for the relatively low velocity diffuse gas molecules. From particle conservation, the diffuse gas flux originating from unit area of the tube wall must be equal to the incoming high velocity gas flux incident on unit area of the wall.

An analytical expression must now be developed for the flux incident on the tube entrance which is also transmitted thru the area of a tube cross-section located at x, some arbitrary distance down the tube. The required function is thus an expression for the flux distribution along the tube length (the high velocity flux which has survived without collisions up to x, as a function of x). The gradient of this flux distribution is the beam flux lost by collisions in unit length of the tube. This is also the source of diffuse gas in unit length of the tube.

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In calculating the flux incident on a probe tube and the flux transmitted directly thru a tube, it is convenient to use a coordinate system which moves with the probe. The undisturbed gas ahead of the entry craft is considered Maxwellian. In this coordinate system, the probe velocity u is superposed on the random molecular velocity, resulting in a velocity biased Maxwellian gas (drifting Maxwellian gas). It is assumed that the velocity u is anti-parallel to the x-axis (zero angle of attack) which is taken to coincide with the longitudinal axis of the probe tube. The velocity biased distribution function may be written

$$f(q) = \frac{n}{\pi^{\frac{3}{2}} v_m^3} e^{-q^2}, \qquad (1)$$

where: $\quad q \equiv \dfrac{w}{v_m}$, $\qquad\qquad\qquad\qquad\qquad (2)$

$\quad n \equiv$ molecular number density in the undisturbed atmosphere

$\quad v_m \equiv$ most probable thermal velocity

$\quad w^2 \equiv (v_x - u)^2 + v_y^2 + v_z^2 \qquad\qquad\qquad (3)$

$\quad v_x, v_y, v_z \equiv$ thermal velocity components in the rest frame of reference.

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Transforming to spherical coordinates, and defining the dimensionless velocity

$$V \equiv \frac{v}{v_m} , \qquad (4)$$

and the speed ratio

$$S \equiv \frac{u}{v_m} , \qquad (5)$$

the distribution function may be written

$$f(V) = \frac{n}{\pi^{3/2} v_m^3} \ e^{-(V^2 - 2VS\cos\phi + S^2)} . \qquad (6)$$

The flux density (flux/unit area normal to the tube axis) incident on the tube entrance from an angle ϕ with the tube axis within $d\phi$, is

$$\psi_1(\phi, S)\, d\phi = \frac{2nv_m}{\pi^{3/2}} \sin\phi \cos\phi\, d\phi \int_0^\pi \int_0^\infty V^3 f(V)\, dV d\theta . \qquad (7)$$

Performing the indicated integrations gives

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$$\psi_1(\phi,S)d\phi = \frac{nv_m}{\sqrt{\pi}}\sin\phi\cos\phi\Big\{(1+S^2\cos^2\phi)\,e^{-S^2} \tag{8}$$

$$+\sqrt{\pi}S\cos\phi\,(\tfrac{3}{2}+S^2\cos^2\phi)(1+erf(S\cos\phi))\,e^{-S^2\sin^2\phi}\Big\}d\phi.$$

The fraction of the flux incident from ϕ on the tube entrance which passes thru a length x of the tube without colliding with the wall is, from the sketch below

$$\psi_2(\phi,X) = \frac{1}{\pi r^2}\int_{A(x)}dA\;. \tag{9}$$

It may be seen from the sketch below that

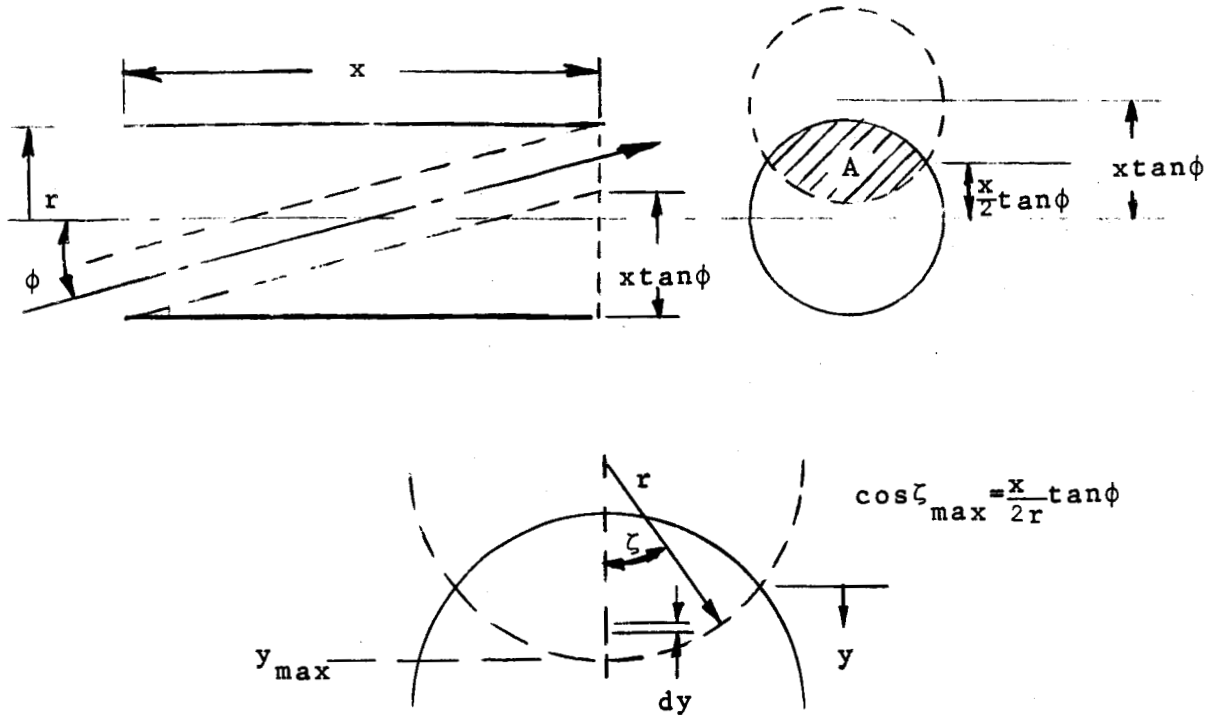$$\int_{A(x)}dA = \int_{o}^{Y_M}4r\sin\zeta\,dy = 4r^2\int_{o}^{\zeta_M}\sin^2\zeta\,d\zeta, \tag{10}$$

or

$$A(x) = 2r^2\Big\{\cos^{-1}(X\tan\phi) - X\tan\phi(1-X^2\tan^2\phi)^{\frac{1}{2}}\Big\}, \tag{11}$$

where
$$X = \frac{x}{2r}. \tag{12}$$

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Therefore, the fraction of the flux incident from ϕ which is directly transmitted thru the tube at least as far as X (considering the possibility of gas-wall collisions only) is

$$\psi_2(\phi,X) = \frac{2}{\pi}\left\{\cos^{-1}(X\tan\phi)-X\tan\phi(1-X^2\tan^2\phi)^{\frac{1}{2}}\right\}. \quad (13)$$

Consider now the effect of gas-gas scattering. If the

tube contains a diffuse gas having a density distribution $n(x)$, the probability that an incident high velocity molecule (which would have reached x with certainty in an empty tube) will now reach x along a trajectory which makes an angle ϕ with the x-axis (without colliding with a diffuse gas molecule) is

$$\psi_3(\phi,X) = e^{-2r\sigma(1+\tan^2\phi)^{\frac{1}{2}}\int_0^X n(X')\,dX'}. \tag{14}$$

This is obtained from the survival equation and the above sketch. The survival equation states that the fraction that remains at x

$$f(x) = e^{-P_c x}, \tag{15}$$

where $P_c \equiv$ collision probability/unit length. For a nonuniform density distribution $n(x)$, the total collision probability up to x is

$$P_c x = \sigma \int_0^x n(x')\,dy, \tag{16}$$

where $dy \equiv$ integration increment along the molecular trajectory which makes an angle ϕ with dx. That is

$$dy = (1+\tan^2\phi)^{\frac{1}{2}}dx, \tag{17}$$

from the above sketch. Normalizing dx by setting

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$$dX = \frac{dx_{real}}{2r} \qquad (18)$$

gives the above result for $\psi_3(\phi,X)$.

Recalling that $\psi_1(\phi,S)$ is the flux density incident from ϕ in $d\phi$, and that $\psi_2(\phi,X)$ is the fraction of the flux which reaches X without colliding with the wall, and that $\psi_3(\phi,X)$ is the probability that a molecule reaches X, along ϕ without colliding with a diffuse gas molecule, then the total flux which reaches X (without encountering a gas-wall or a gas-gas collision) is

$$F(S,X) = \pi r^2 \int_0^{\tan^{-1}X^{-1}} \psi_1(\phi,S)\,\psi_2(\phi,X)\,\psi_3(\phi,X)\,d\phi \; . \qquad (19)$$

This is the relation required to determine the density in the undisturbed atmosphere from mass analysis measurements of the sampler exit beam which enters the analyzer. No mathematical approximations have been made in deriving this relation. This integral cannot be evaluated analytically. It can, of course, be evaluated numerically. Eventually, it will be required that a value for $F(S,X)$ be known for all X such that $0 \leq X \leq X_{max}$, where X_{max} is the actual length to diameter ratio of the probe tube. For each required $F(S,X)$,

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a new value must be calculated from the integral expression for $F(S,X)$ with the upper limit changed to the appropriate value of X.

It is sometimes convenient to have a dimensionless expression for the fraction of the flux incident on the tube entrance which is transmitted thru the tube exit, unscattered. This transmission function is given by

$$T(S,X) \equiv \frac{F(S,X)}{F(S,O)} , \tag{20}$$

where both numerator and denominator are obtained from the above integral expression, evaluated at the appropriate limits.

In the above integral expression for the high velocity flux transmitted thru the tube, the integrand depends on the integral over the diffuse gas density distribution in the tube, as may be seen from $\psi_3(\phi,X)$. The equilibrium density distribution within the tube may be determined by applying the diffusion equation to the diffuse gas generated within the tube, by providing a gas source term in the equation which corresponds to the high velocity stream loss (particle conservation). The applicable form of the diffusion equation is

$$\frac{\pi^2 r^3 \bar{v}}{4} \frac{d^2 n(x)}{dx^2} = \frac{dF(S,x)}{dx} , \tag{21}$$

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where $n(x) \equiv$ diffuse gas density at x, and $\bar{v} \equiv$ mean thermal velocity of the diffuse gas in the tube. The flux gradient of the high velocity stream on the right is the magnitude of the loss from that stream in unit length of the tube. This loss is the result of both gas-wall and gas-gas scattering within the tube. In both collision processes, it is assumed that a high velocity molecule which encounters the wall or a diffuse gas molecule is immediately removed from the high velocity stream and converted to a diffuse gas molecule. Thus, the term on the right above is also the source of diffuse gas in unit length. The boundry conditions appropriate to the above equation in this application are that the diffuse gas density distribution within the tube approaches zero at the tube entrance and at the tube exit. This corresponds to the physical assertion that the diffuse gas density outside the tube is sufficiently low that back diffusion may be neglected. Under these conditions the solution to the above equation is

$$n(X) = \eta \left\{ \int_0^X F(S,X) dX - (X/X_M) \int_0^{X_M} F(S,X) dX \right\} \qquad (22)$$

where
$$\eta \equiv \frac{8}{\pi^2 r^2 \bar{v}} \qquad (23)$$

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and $\bar{v} \equiv$ mean thermal velocity of the diffuse gas in the tube, assumed to be in thermal equilibrium with the tube.

At this point an expression has been developed for the high velocity flux distribution in the tube which depends on the diffuse gas density distribution in the tube, and an expression for the diffuse gas density distribution has been developed which depends on the high velocity flux distribution in the tube. These two equations, the above equation for $n(X)$ and the previous integral equation for $F(S,X)$, must be solved as a system. An efficient way to solve this system is by successive iteration. This procedure begins by considering an empty tube in which $n(X) = 0$. Then obtaining a 1st approximation for the high velocity flux distribution $F_1(S,X)$. This solution accounts for gas-wall scattering only. $F_1(S,X)$ is then inserted in the expression for $n(X)$ and a 1st approximation for the diffuse gas density distribution in the tube, $n_1(X)$ is obtained. The procedure is then repeated and a 2nd approximation for the flux $F_2(S,X)$ is obtained by using $n_1(X)$ in the integral expression for $F(S,X)$ and then using $F_2(S,X)$ to calculate a 2nd approximation to the diffuse gas density distribution, $n_2(X)$. The process is repeated until successive iterations yield results which do not differ by

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more than a prescribed amount corresponding to the desired accuracy of the final solution.

These results are exact in the sense that no mathematical approximations have been introduced, the only approximation in the entire system is in the molecular collision model used and consists of the assumption that complete energy accommodation occurs in collisions involving high velocity molecules. While this is not a good approximation to existing experimental data on high velocity molecular scattering, it is conservative in the sense that it leads to a substantial overestimate of the diffuse gas density in the sampler tubes which necessarily overestimates the attenuation of the transmitted beam by gas-gas scattering and therefore substantially underestimates the maximum density in the undisturbed atmosphere at which the sampler can operate with precision. A possible alternate model which does not require this assumption is discussed later.

In Appendix A some analytical approximations to the above results have been worked out which may be used, under certain conditions, to estimate the sampler unscattered exit flux and other gas dynamic parameters with relatively modest computational labor.

Calculations based on these analytical results imply that the unscattered exit beam from the sampler is linearly

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related to the undisturbed atmospheric density down to about 39 Km (118000 ft) above the Mars surface (using VM8 as the model atmosphere and taking the entry craft velocity as 3.94 Km/sec (12000 ft/sec)).

3.2 Thermal Flux Analysis

The molecular flux incident on the sampler has a high enthalpy relative to the sampler. Since the entry craft speed is large compared to the mean thermal velocity of the undisturbed atmosphere, the molecular enthalpy, relative to the probe, is substantially

$$H = \frac{1}{2}mu^2, \tag{24}$$

where $m \equiv$ molecular mass, and $u \equiv$ entry craft speed. The PV term has been neglected since the gas pressure is negligible. Since the molecular flux which is transmitted thru the tubes unscattered, delivers no energy to the sampler, the average number of molecules incident on unit projected area of the sampler in unit time, is

$$\frac{\dot{N}}{A_\perp} = \frac{F(S,0)-F(S,X_M)}{\pi r^2} \simeq nu[1-T(S,X_M)], \tag{25}$$

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where $n \equiv$local, undisturbed molecular number density.

Therefore the average power density incident on the sampler, associated with the enthalpy of the incident molecular flux is

$$w_i = \frac{1}{2}nmu^3[1-T(S,X_M)] = \frac{1}{2}\rho_m u^3[1-T(S,X_M)] \tag{26}$$

where $\rho_m \equiv$undisturbed local mass density. It has again been conservatively assumed that the energy accommodation coefficient is unity. The incident power density is not uniform along the sampler tube, but the physical dimensions of the tube are so small that the power density may be considered uniform with negligable error.

The principal mode of heat loss from the sampler tubes is by radiation. The radiation power density is

$$w_r = \varepsilon\sigma T^4, \tag{27}$$

where $\varepsilon \equiv$effective emissivity, and $\sigma \equiv$Stefan-Boltzmann constant. The effective emissivity of a bundle of tubes having small diameter to length ratio is approximately 1. Both ends of the tube bundle radiate equally and it is probable that most of the side wall of the bundle will be used as structural mounting. Therefore the equilibrium temperature of the tube

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bundle is that for which the total power radiated from both ends of the bundle equals the incident power. From the above equations then

$$2\sigma T^4 = \frac{1}{2}\rho_m u^3 [1 - T(S, X_M)].$$
(28)

Taking 1500°K as the maximum allowable sampler temperature, and the entry craft velocity as $u = 3.9$ Km/sec, and $T(S, X_M) \simeq .5$, gives a maximum mass density (at which the sampler can operate without exceeding its temperature limitation) of about 4.2×10^{-8} gm/cm^3. This corresponds to a height of about 38 Km(116,000 ft.) for model atmosphere VM-8.

Some appreciation of the rate of change of this minimum operating altitude with maximum allowed temperature may be obtained by taking the sampler temperature limit to be 1700°K. If the other parameters are held fixed the minimum operating height is then 35Km(106,000 ft).

These numbers are probably excessively conservative since they are based on an energy accommodation coefficient of unity, and high velocity scattering data implies that the actual value of the energy accommodation coefficient is much nearer zero than unity for low angles of incidence. For the high velocity flux incident on the tube walls at the low angles of incidence under consideration here, an energy accommodation coefficient of 0.1 is probably more realistic. The

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last equation should then be written

$$2\sigma T^4 = \frac{1}{2}\rho_m u^3 [1-T(S,X_M)]\bar{\alpha}_E , \qquad (29)$$

where $\bar{\alpha}_E \equiv$ energy accommodation coefficient (average). Taking $\bar{\alpha}_E = 0.1$ and again taking 1500°K as the maximum operating temperature and $u = 3.9$ Km/sec, and $T(S,X_M) \simeq .5$, the minimum operational altitude for the sampler is 27Km(82,000 ft).

3.3 Continuum Flow Effects

The altitude at which continuum flow effects, such as as shocks and pressure gradients become important may be estimated by calculating the density at which the mean free path in the undisturbed atmosphere is comparable to the diameter of the sampler tubes. The maximum number density at which the sampler may be operated is then

$$n = \frac{1}{2\sqrt{2}\sigma r} , \qquad (30)$$

where $\sigma \equiv$ collision cross-section, $(4\pi R^2)$,
$R \equiv$ molecular radius, and $r \equiv$ tube radius.

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Taking VM-8 as the model atmosphere and the tube radius $r = 5 \times 10^{-3}$cm, then the minimum height at which the sampler is operable with respect to continuum flow constraints, is about 19Km(58,000 ft). It is of course, assumed that in the design of the sampler support and mount, proper weight is given to the need for maintaining the support diameter small at the point of attachment to the sampler. It is also assumed that the support structure protrudes sufficiently far forward of the entry craft that the sampler is clear of any shock waves associated with the entry craft, which will appear at substantially higher altitudes.

3.4 Pumping Requirements

Since the entry craft velocity is much greater than the mean thermal velocity of the diffuse gas which accumulates in the sampler support structure, an aft-facing exhaust port is an efficient pump. The gas inside may diffuse out, but only a negligable fraction of the molecules outside have velocity components such that they can enter the port. Since the gas flux incident on the sampler inlet has a much higher velocity than the mean velocity of the diffuse gas effusing overboard thru the exhaust ports, the effective conductance

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of the exhaust system must be larger by many orders of
magnitude than that of the sampler if the diffuse gas den-
sity within the support structure is to be maintained suf-
ficiently low that attenuation of the transmitted beam by
gas-gas scattering is negligable. It turns out that this
requirement can be well satisified, since the inlet area
is quite small.

The use of several beam limiting apertures divides the
volume of the sampler support into several compartments.
Each compartment must be provided with an exhaust port. The
diffuse gas component effusing from the sampler exit is
distributed over a much larger angle than the transmitted
beam. The combination of limiting apertures and exhaust
ports thus minimize the flux of diffuse gas entering the
analyses compartment. Based on existing data for the
angular distribution of the diffuse flux from the sampler
exit and an analytic estimate of the diffuse gas density
distribution within the sampler tubes, it is expected that
the ratio of diffuse flux to unscattered flux entering
the analysis compartment will be less than a few percent.

4. CONCLUSIONS

A number of general classes of gas sampling systems
have been reviewed with respect to their applicability and
practicability for use on a high speed entry craft to in-
vestigate the atmosphere of Mars. It is concluded that a
sampler consisting of a bundle of small diameter tubes hav-
ing a moderate length-to-diameter ratio, which directs an
unscattered molecular beam into the analyzer, has the high-
est probability of obtaining an exact sample of the undis-
turbed atmosphere and maintaining it unaltered and uncon-
taminated until delivered to the analyzer.

A theoretical model has been developed, to describe
in analytical terms, the high velocity, free molecular flow
thru the sampler. In particular, the theory yields an ex-
pression for the unscattered exit flux from the sampler.
This relation may be used to interpret the measured mass
spectrum of the unscattered beam in terms of the relative
abundance and absolute molecular density in the undisturbed
atmosphere. The relation is exact over most of the useful
range of operation and becomes only slightly inexact near
the low altitude limit of operation.

Using the theory that has been developed, taking a

tube diameter of 10^{-2} cm as the present state of the art, and
taking VM-8 as the model atmosphere and 3.94 Km/sec (12000
ft/sec) as the entry velocity, the altitude down to which the
sampler performs its function satisfactorily has been cal-
culated with respect to the principal limiting effects. In
the table below, the important limiting effects are listed
on the left and the altitudes below which they begin to
limit the performance of the sampler are tabulated in the
middle column. Since the theoretical model used is conserv-
ative, a more realistic estimate is given in the right column
of the probable limiting altitudes (based on a judgment of
the magnitude of conservatism inherent in the scattering
model used).

EFFECT	ALTITUDE LIMIT (Km)	
	THEORY (CONSERVATIVE)	PROBABLE
Gas-gas scattering	39	less than 30
Gas enthalpy heating	38	less than 25
Near FMF Limit	19	less than 19

All these limits may be lowered by decreasing the size
of the sampler tubes, however, the gas enthalpy heating
limit is rather insensitive to this action.

It is concluded from these results that the unscattered
molecular beam-forming sampler performs its function correctly
and satisfactorily over the major part of the entry trajec-
tory and comparative evaluation indicates that it is substan-
tially superior in performance to other techniques investigated.

5.   RECOMMENDATIONS

1.   Since the theoretical model developed in this inves-
tigation for the unscattered exit flux from the sampler is
found to be very useful in predicting the dynamic perform-
ance of the sampler and the performance range of the sampler,
and since it appears that the performance range will be
limited in practice by gas-gas scattering within the sampler
tubes, it is recommended that the study be continued along
a similar course but include a more accurate treatment of
the energy accommodation coefficient.

It is expected that a specular reflection model, rather
than the diffuse reflection model used in this investigation,

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will yield a more accurate prediction of the useful range (limiting altitude) of the sampler and give a more accurate relation between the measured mass analysis and the molecular density in the undisturbed atmosphere near the limit of operation.

2. It is recommended that the problems connected with design, fabrication, and assembly of a sampler be investigated by building a prototype sampler using the smallest tube now available in any satisfactory material. During execution of this work, particular attention should be given to evaluating the practicability of using even smaller tubes in the sampler.

3. There are a number of problems connected with the performance of the sampler which do not appear to be altogether amenable to analytical treatment. These include such problems as evaluating the temperature distribution within the sampler and the changes in temperature distribution as a function of small, abrupt changes in angle of attack, under the influence of a high enthalpy flux density. An important related question concerns the effect that changes in temperature distribution may have on dimensional stability and the internal stress distribution. Even the maximum allowable operating temperature for a given material cannot be established with satisfactory confidence in such

a complex structure, based only on theory. Further, the structural stability of the sampler at or near its maximum operating temperature, under dynamic entry conditions at relatively low altitude, with a time varying angle of attack and temperature distribution, is a formidable problem, probably more amenable to assault by experiment than theoretical analysis.

It is considered preferable to attack these and similar problems experimentally, since only under experimental conditions closely approximating the anticipated flight conditions can practical solutions be developed with high confidence. The most direct and efficient experimental approach is to test, evaluate, and study the performance of a sampler in the high density flux of a "hot" molecular beam. (Hot, as used here, implies an effective total temperature of the order of $50,000°K$). It is therefore recommended that the feasibility of using a hot molecular beam to experimentally evaluate the sampler performance, limitations and behavior under realistic conditions be investigated.

APPENDIX A

UNSCATTERED FLUX APPROXIMATE ANALYSIS

For $1 << X << S$, the unscattered exit flux $F(S,X)$ may be approximated analytically. This condition implies that

$$\cos \phi \simeq 1$$
$$\tan \phi \simeq \phi$$
$$\sin \phi \simeq \phi \qquad \qquad \text{(A1)}$$
$$\tan^{-1} X^{-1} \simeq X^{-1}$$

and,

$$[1 + \tan^2 \phi]^{\frac{1}{2}} \simeq 1 \;,$$

from which it follows that

$$\tilde{\Psi}_1(\phi,S) = \frac{nv_m}{\sqrt{\pi}} \; \phi \left\{ (1+S^2) \; e^{-S^2} + \sqrt{\pi} S (\frac{3}{2}+S^2)(1+\text{erf}(S)) \; e^{-S^2\phi^2} \right\}, \text{(A2)}$$

and

$$\tilde{\Psi}_2(\phi,X) = \frac{2}{\pi} \left\{ \cos^{-1}(X\phi) - X\phi(1-X^2\phi^2)^{\frac{1}{2}} \right\} \;, \qquad \text{(A3)}$$

and

$$\tilde{\psi}_3(\phi,X) = e^{-2r\sigma\int_0^X n(X')dX'}. \tag{A4}$$

Under the above condition $\tilde{\psi}_3$ is independent of ϕ. $\tilde{\psi}_1(\phi,S)$ may be simplified further, since for large S, both e^{-S^2} and $S^2 e^{-S^2}$ are negligable compared to 1, and $\text{erf}(S) \simeq 1$. Also, the factor $(\frac{3}{2}+S^2)$ may be replaced with S^2 without introducing a substantial error. Therefore, $\tilde{\psi}_1(\phi,S)$ may be written

$$\tilde{\psi}_1(\phi,S) = 2nv_m S^3 e^{-S^2\phi^2}\phi. \tag{A5}$$

Combining these approximations, the unscattered exit flux may be written

$$\tilde{F}(S,X) = 2\pi r^2 nv_m S \left\{ e^{-2r\sigma\int_0^X n(X')dX'} \right\}$$

$$\cdot \frac{2}{\pi} \int_0^{X^{-1}} e^{-S^2\phi^2} S^2 \left\{ \cos^{-1}(X\phi) - X\phi[1-X^2\phi^2]^{\frac{1}{2}} \right\} \phi d\phi \tag{A6}$$

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Finally, the analytical approximation for $\tilde{F}(S,X)$ may be written (after integrating by parts)

$$\tilde{F}(S,X) = \pi r^2 n v_m S\, e^{-2r\sigma \int_0^X n(X')dX'} \left\{ 1 - \frac{4}{\pi} \int_0^1 e^{-Z^2 y^2}(1-y^2)^{\frac{1}{2}}dy \right\} , \quad (A7)$$

where
$$Z = SX^{-1}, \quad (A8)$$
and
$$y = X\phi. \quad (A9)$$

The total flux incident on the tube entrance may be evaluated exactly

$$F(S,O) = \pi r^2 \int_0^{\pi/2} \psi_1(\phi,S)d\phi = \frac{\pi r^2}{2}\left(\frac{nv_m}{\sqrt{\pi}}\right)\left\{ e^{-S^2} + \sqrt{\pi}S[1+\mathrm{erf}(S)] \right\}. \quad (A10)$$

But for $S \gg 1$, $F(S,O)$ may be written approximately

$$\tilde{F}(S,O) \simeq \pi r^2 n v_m S \quad (A11)$$

Therefore the analytical approximation for the fraction of

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the flux transmitted through the tube (without gas-wall
or gas-gas scattering) is

$$\tilde{T}(S,X) \equiv \frac{\tilde{F}(S,X)}{\tilde{F}(S,O)} \quad \text{(Transmission Function)} \quad (A12)$$

$$= e^{-2r\sigma\int_o^X n(X')dX'}\left\{1-\frac{4}{\pi}\int_o^1 e^{-Z^2y^2}(1+Y^2)^{\frac{1}{2}}dy\right\} .$$

For large S, and an empty tube, the numerical evalua-
tion of the integral expression for F(S,X) for a fixed S
and a number of values of X within the range $0 \leq X \leq 10$ yields
data for T(S,X) which may be approximated by an exponential
function of the form

$$T_S(X) = e^{-\beta X} . \quad (A13)$$

Since it is the integral of $T_S(X)$ which is of interest here,
this is a satisfactory approximation. β is evaluated by
fitting this function to the analytical approximation for
the transmission function above, at X_M

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$$T_S(X_M) = T(S, X_M) \ , \tag{A14}$$

from which it follows that

$$e^{-\beta X_M} = 1 - \frac{4}{\pi} \int_0^1 e^{-Z^2 y^2} (1-y^2)^{\frac{1}{2}} dy, \tag{A15}$$

$$\text{evaluated at} \quad Z = SX_M^{-1}. \tag{A16}$$

Now that β has been evaluated, the integrable approximation for the transmission function, Eq(A13), may be substituted into Eq(22) to obtain the diffuse gas density distribution within the tube

$$\tilde{n}(X) = \eta \tilde{F}(S,0) \left\{ \int_0^X T_S(X') dX' - X/X_M \int_0^{X_M} T_S(X) dX \right\} \tag{A17}$$

where

$$\eta = \frac{8}{\pi^2 r^2 \bar{v}} \ , \tag{A18}$$

and

$$\tilde{F}(S,0) = \pi r^2 nu. \tag{A19}$$

Substituting for $T_S(X)$ from Eq. (A13) and integrating, gives

$$\tilde{n}(X) \simeq \frac{8nu}{\pi \bar{v}} \left\{ \frac{1-e^{-\beta X}}{\beta} - \frac{X}{X_M} \cdot \frac{1-e^{-\beta X_M}}{\beta} \right\}. \qquad (A20)$$

This diffuse gas density distribution within the tube results from the flux incident on the tube entrance which is scattered off the tube wall. The 1st order effect of the diffuse gas within the tube is to attenuate the transmitted flux by gas-gas scattering. Rather than calculate the magnitude of the attenuation, it is more useful to calculate the density which produces a prescribed attenuation. Suppose that the exit beam attenuation by gas-gas scattering is not allowed to exceed 10%. Then

$$0.9 = e^{-2r\sigma \int_0^{X_M} n(X)dX}. \qquad (A21)$$

Integrating $\tilde{n}(X)$ and taking the logarithm gives

$$\frac{16r\sigma nu}{\pi \bar{v}} \frac{1}{\beta} \left\{ \frac{X_M}{2} - \frac{1}{\beta} + \left(\frac{X_M}{2} + \frac{1}{\beta} \right) e^{-\beta X_M} \right\} \simeq 10^{-1}. \qquad (A22)$$

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For $X_M = 5$, and using Eq(A15) evaluated at $Z = 3.5$ $\quad$ (A23) gives

$$\frac{1}{\beta} \left\{ \frac{X_M}{2} - \frac{1}{\beta} + \left(\frac{X_M}{2} + \frac{1}{\beta} \right) e^{-\beta X_M} \right\} = .526 \qquad \text{(A24)}$$

thus

$$n = \frac{\pi}{(16)(5.26)} \frac{\bar{v}}{u} \frac{1}{r\sigma} , \qquad \text{(A25)}$$

and taking

$$\sigma = \pi (3.24 \times 10^{-8})^2 = 3.3 \times 10^{-15} \text{cm}^2 , \qquad \text{(A26)}$$

$$u = 3.94 \times 10^5 \text{cm/sec} , \qquad \text{(A27)}$$

$$\bar{v} \big|_{\substack{T = 1500 \\ M = 44}} = .855 \times 10^5 \text{cm/sec} \qquad \text{(A28)}$$

$$r = 5 \times 10^{-3} \text{cm} , \qquad \text{(A29)}$$

it then follows that the maximum allowed molecular density in the undisturbed atmosphere is

$$n = 4.92 \times 10^{14} cm^{-3}. \qquad (A30)$$

Using VM-8 as the model atmosphere, the corresponding mass density is

$$\rho_m \Big|_{M \,=\, 44} = 3.62 \times 10^{-8} gm/cm^3, \qquad (A31)$$

which corresponds to a height of about 39 Km.

The maximum density in the tubes may be evaluated by solving $\dfrac{dn(X)}{dX} = 0$ for X and substituting this value of X back into n(X). Defining $\hat{X}$ as the value of X at which n(X) passes thru its maximum, it follows that

$$\frac{dn(X)}{dX} = 0 = e^{-\beta \hat{X}} - \frac{1 - e^{-\beta X_M}}{X_M \beta} \qquad (A32)$$

or

$$e^{-\beta \hat{X}} = \frac{1 - e^{-\beta X_M}}{X_M \beta}. \qquad (A33)$$

Substituting this value for $\hat{X}$ into $\tilde{n}(X)$ gives

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$$\tilde{n}(X)\bigg|_{Max} = \tilde{n}(\hat{X}) = (.243)\frac{8}{\pi}\, n\, \frac{u}{\bar{v}} \quad . \tag{A34}$$

Using the values from Eqs. (A27), (A28) and (A30) for u, $\bar{v}$, and n gives for the maximum diffuse gas density in the tube

$$\tilde{n}(\hat{X}) = 1.89 \times 10^{15}\, cm^{-3}. \tag{A35}$$

If the diffuse gas mean free path, evaluated at this maximum diffuse density in the tube, is small compared to 2r, then the diffuse gas denisty distribution within the tube is controlled by free-molecular-flow diffusion. The minimum mean free path of a gas molecule in equilibrium with the tube temperature is

$$\lambda_{min} = \frac{1}{\sqrt{2}\sigma n(\hat{X})} \quad , \tag{A36}$$

and using the values previously determined, gives

$$\lambda_{min} = 0.114\ cm \quad ,$$

and

$$\frac{\lambda_{min}}{2r} = 11.4.$$

Therefore, λ_{min} could be decreased by about an order
of magnitude ($\tilde{n}(\hat{X})$ increased by an order of magnitude)
before continuum flow effects become important.